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SYNTHESIS OF OPTIMUM FIN-LINE TAPERS

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Summary

The theory of TEM matching sections is modified and applied to fin-line tapers. For the VSWR prescribed, the minimum length taper contour is derived. Measurements confirm the synthesis technique.

TAPER SYNTHESIS FOR TEM STRUCTURES

For TEM structures, the design of optimum tapers is well-known from literature. Their input reflection coefficient reads /1/

$$R(\beta)=-\int_0^l \varkappa^{-+}(z')e^{-2j\beta z'}dz' \qquad (1)$$

with $\varkappa^{-+}(z)$ z-dependent coupling coefficient between reflected and incident fundamental mode, l taper length, and $\beta=2\pi f\sqrt{\varepsilon\mu}$ propagation constant. From (1), $\varkappa^{-+}(z)$ can be deduced by Fourier transformation. Solutions are known keeping the input reflection coefficient R below a certain value R_{max} for $\beta>\beta_o$, so that high-pass performance is obtained.

TAPER SYNTHESIS FOR NON-TEM STRUCTURES

According to /2/, an expression equivalent to (1) can be found for non-TEM waves, if the coupling coefficient is approximated by $\varkappa^{-+}(f,z)\approx\varkappa^{-+}(f_o,z)$ at a fixed frequency f_o. Furthermore we approximate the integral

$$\int_0^z 2\beta(z')dz'\approx\eta(f)\cdot\xi(z) \qquad (2)$$

by a product of a purely frequency dependent and a purely z-dependent factor. $\eta(f)$ is normalized so that $\eta(f_o)=1$. This results in

$$R(\eta)=\int_{-\Theta}^{\Theta} CK(\xi)e^{-j\eta\xi}d\xi \text{ with } \Theta=\xi(1)=-\xi(0) \text{ and} \qquad (3)$$

$$C\cdot K(\xi)=-\varkappa^{-+}(f_o,z)/(2\beta(f_o,\xi)). \qquad (4)$$

The integral(3) is of the same type as (1), so that the coupling distributions $C\cdot K(\xi)$ known from TEM -theory can be applied.

SYNTHESIZING THE CUTOFF FREQUENCY FUNCTION

Once $\varkappa^{-+}(f_o,\xi)$ has been related to $\beta(f_o,\xi)$, the latter can be evaluated from $CK(\xi)$. Such a relation can be derived, if we neglect both the hybrid character of the fin-line fundamental mode and the longitudinal component of the magnetic field along the thickness of the fin metallization. The first assumption is valid for thin substrates with low ε_r, which are widely used, the second requires that the substrate is not too far off the center of the waveguide. Both approximations hold the better the narrower the slot width. Thus β and $\varkappa^{-+}$ can be expressed with the cutoff frequency as parameter /2/:

$$\beta=2\pi f\sqrt{\varepsilon_o\mu_o}\sqrt{k_e}\sqrt{1-(f_c/f)^2}, \qquad (5)$$

$$\varkappa^{-+}=-\frac{1-(f_c/f)^2/2}{1-(f_c/f)^2}\cdot\frac{1}{f_c}\cdot\frac{df_c}{dz}. \qquad (6)$$

k_e is the effective dielectric constant of the fin -line.

DISTRIBUTION OF THE COUPLING COEFFICIENT

The shortest taper with $|R|<R_{max}$ for $f>f_o$ follows from the Dolph-Tschebyscheff distribution /3/, which is characterized by

$$K(\xi)=\frac{D}{2}\left\{\frac{I_1(\Theta\sqrt{1-(\xi/\Theta)^2})}{\sqrt{1-(\xi/\Theta)^2}}+\delta(\xi-\Theta)+\delta(\xi+\Theta)\right\} \qquad (7)$$

for $-\Theta\le\xi\le\Theta$ with $D=R_{max}/C$, Θ = arccosh (1/D), $I_1(x)$ the modified Bessel function of the first order, and $\delta(t)$ Dirac's delta function. C is a normalizing constant according to

$$\int_{-\Theta}^{\Theta}K(\xi)d\xi=1. \qquad (8)$$

Integrating (4) from $\xi= -\Theta$ to Θ with (6) yields

$$C=\frac{1}{4}\ln\left[\left(\frac{f_c(0)}{f_c(1)}\right)^4\cdot\frac{1-(f_c(1)/f_o)^2}{1-(f_c(0)/f_o)^2}\right]. \qquad (9)$$

Integrating (4) from $-\Theta$ to ξ results in the ξ-dependent cutoff frequency

$$f_c(\xi)=f_c(0)\cdot\left[F/2+\sqrt{F^2/4+(1-F)\cdot\exp(4CI(\xi))}\right]^{-1/2} \qquad (10)$$

with $F=(f_c(0)/f_o)^2$ and $I(\xi)=\int_{-\Theta}^{\xi}K(\xi')d\xi'$. Due to the normalization (8) and $K(\xi)=K(-\xi)$ we can evaluate the integral:

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$$I(\xi)=\frac{1}{2}+\frac{D\Theta^2}{2}\int_0^{\xi/\Theta}\frac{I_1(\Theta\sqrt{1-y^2})}{\Theta\sqrt{1-y^2}}\,dy \quad \text{for } |\xi|<\Theta,$$

$$=0 \text{ for } \xi=-\Theta, \text{ and } =1 \text{ for } \xi=\Theta. \qquad (11)$$

The Bessel function in (11) can be expanded into a power series /4/, so that the integral can be integrated term by term. The series converges rapidly. The relative error is less than 10^{-5} for truncation behind the 12th term.

Dirac's function in (7) causes discontinuities at the taper ends, which might excite higher-order modes. The step in cutoff frequency can be calculated from (9), (10) and (11):

$$f_c(+0)/f_c(0)=1-R_{max}(f_o-f_c(0))/(2f_o-f_c(0)), \quad (12)$$

The discontinuity at the taper end with the narrow slot width is still smaller. The step is the smaller the lower R_{max} is chosen. For $R_{max}=0.01$ ($\hat{=}$ -40 dB) and $f_o = 1.4\, f_c(0)$, the ratio in (12) is 0.997. A deterioration of the performance by this small step could not be observed experimentally.

SYNTHESIZING THE SLOT CONTOUR

We are now able to synthesize the function of the cutoff-frequency f_c along the taper. (It should be noted that no wave impedances are required for this synthesis, so that difficulties in defining an appropriate impedance are circumvented.) Hence we need a relation between slot width and cutoff frequency which should be easy for evaluation because it is called at every knot on the z-axis. The cutoff-frequency has been calculated from a transverse resonance condition, which can be evaluated for the slot width as well as for the cutoff frequency. A suitable formula for the slot capacitance was found /5/, which is valid for slot widths up to the waveguide height. The slot may be located unsymmetrically. Evaluating the resonance condition with and without dielectric gives a value for the effective dielectric constant k_e.

The synthesis procedure can now be summarized as follows: 1. Prescribe f_o, R_{max}, and longitudinal step width Δz; choose an appropriate relation between slot width s and eccentricity e (see insert in Fig. 2) in order to obtain a unique function $s(f_c)$, e.g.

$$e(s(z))=e(0)+(e(1)-e(0))(s(z)-s(0))/(s(1)-s(0)). \quad (13)$$

2. Determine $f_c(0)$ and $f_c(1)$ from s(0) and s(1) and the transverse resonance condition.
3. Determine the normalization factor C from (9).
4. Set initial values z=0, $\xi=-\Theta$ and s = s(0).
5. Take next $z:=z+\Delta z$ and evaluate $\xi(z)$ from (2) as $\xi:=\xi+2\beta(f_o,z)\Delta z$, f_c from (10), and s from the transverse resonance condition.
6. Repeat step 5 until the final slot width is reached.

EXPERIMENTAL RESULTS

Knowing $s(f_c)$ optimum tapers can be realized. Two tapers between 0.2 mm slot width and standard waveguide have been mounted back to back. Measurement sensitivity was limited to -40 dB for the reflection coefficient. Results are shown in Figs. 1-3. Taper length in Ku-band is only 18 mm for $R_{max}<$ -30 dB. The contour function must be realized with an accuracy of 20 µm (40 µm) for a return loss of less than -30 (-20) dB.

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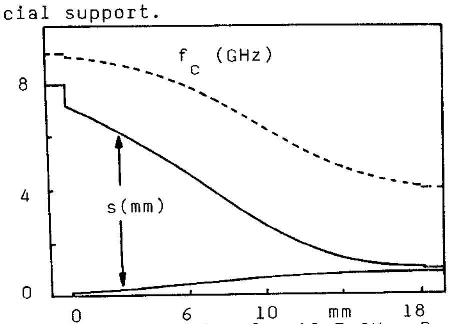

Fig. 1 Taper contour for f_o= 12.3 GHz, R_{max}=-30 dB

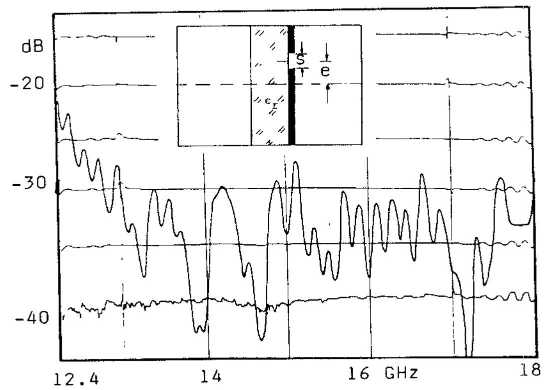

Fig. 2 Return loss of double taper of Fig. 1

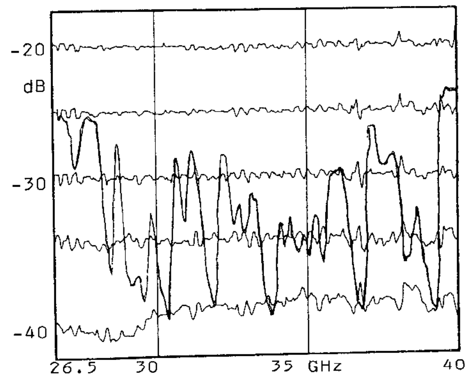

Fig. 3 Double taper, f_o=26 GHz, R_{max}=-30 dB, length 8.2 mm.